MAT 261—Exam #1A—2/18/14

Name: _____

Calculators are not permitted. Show all work using correct mathematical notation.

1. (10 points) Find the slope of the tangent line to the curve $c(t) = (4 \ln t, t^3 + 5)$ at the point where $t = 2$. Give your answer in simplest possible form.

2. (15 points) A particle moves in space with trajectory

$$x(t) = e^{3t}, \quad y(t) = t^2 \sin \pi t, \quad z(t) = \frac{4}{(t+1)^2}.$$

Find the speed of the particle at $t = 1$. Give your answer in simplest possible form.

3. (13 points) Find the angle between the vectors $\mathbf{v} = \mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $\mathbf{w} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$. You may express your answer in terms of inverse trigonometric functions.

4. (12 points) Find the area of the parallelogram determined by the vectors $\mathbf{v} = 3\mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\mathbf{w} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$.

5. (10 points) Find parametric equations for the line that is perpendicular to the plane $x + 2y + 4z = 8$ and passes through the point $(7, 5, 3)$.

6. (15 points) Find a unit vector perpendicular to the plane containing the points $P(2, -1, 0)$, $Q(1, 0, 1)$, and $R(0, 3, -1)$.

7. (10 points) Convert the spherical coordinates $(\rho, \theta, \phi) = (\sqrt{2}, \pi/3, \pi/4)$ into rectangular coordinates (x, y, z).

8. (15 points) Find the length of the helix defined by

$$\mathbf{r}(t) = (\cos 5t)\mathbf{i} + (\sin 5t)\mathbf{j} + 2t^{3/2}\mathbf{k} \qquad (0 \leqslant t \leqslant 1).$$