MAT 162—Exam #2—10/24/13

Name: Solutions

Show all work using correct mathematical notation. Calculators are not permitted.

1. (15 points) Evaluate $\displaystyle\int_0^{\pi/2} \cos^3 x \, dx$.

$$= \int_0^{\pi/2} (1 - \sin^2 x) \cos x \, dx$$

$u = \sin x$

$du = \cos x \, dx$

$$= \int_0^1 (1 - u^2) \, du$$

$$= u - \frac{u^3}{3} \Big|_0^1$$

$$= \frac{2}{3}$$

2. (10 points) Evaluate $\displaystyle\int \frac{2x+1}{x^2+25} \, dx$.

$$= \int \frac{2x}{x^2+25} \, dx + \int \frac{1}{x^2+25} \, dx$$

$u = x^2 + 25$

$du = 2x \, dx$

$$= \int \frac{du}{u} + \frac{1}{5} \tan^{-1}\left(\frac{x}{5}\right)$$

$$= \ln(x^2+25) + \frac{1}{5} \tan^{-1}\left(\frac{x}{5}\right) + C$$

3. (15 points) Evaluate $\int xe^{3x}\,dx$.

$u = x \qquad dv = e^{3x}$

$du = dx \qquad v = \frac{1}{3}e^{3x}$

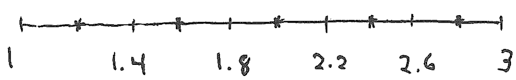
$$\int x e^{3x}\,dx = \frac{1}{3} x e^{3x} - \int \frac{1}{3} e^{3x}\,dx$$

$$= \frac{1}{3} x e^{3x} - \frac{1}{9} e^{3x} + C$$

4. (10 points) Consider the integral $\int_1^3 x^3\,dx$.

(a) Use the Midpoint Rule with $N = 5$ to approximate the integral. Just write out the terms in your sum–do not attempt to add them up.

$$\Delta x = \frac{3-1}{5} = 0.4$$



$$M_5 = 0.4\left((1.2)^3 + (1.6)^3 + 2^3 + (2.4)^3 + (2.8)^3 \right)$$

(b) Determine the maximum possible error in your estimate from part (a).

$$f(x) = x^3 \Rightarrow f'(x) = 3x^2 \Rightarrow f''(x) = 6x$$

So $|f''(x)| \leq 6 \cdot 3 = 18$ on $[1,3]$

Thus $$|E_M| \leq \frac{18\,(3-1)^3}{24 \cdot 5^2} = \frac{6}{25}$$

5. (10 points) Evaluate the improper integral $\int_4^\infty \frac{1}{x^{3/2}}\,dx$.

$$\int_4^\infty \frac{1}{x^{3/2}}\,dx = \lim_{b\to\infty} \int_4^b x^{-3/2}\,dx$$

$$= \lim_{b\to\infty} -2x^{-1/2}\Big|_4^b$$

$$= \lim_{b\to\infty} \left(\frac{-2}{\sqrt{b}} + \frac{2}{\sqrt{4}} \right)$$

$$= 1$$

6. (15 points) Evaluate $\int \frac{2x^3+5x^2+15x+5}{x^2+x}\,dx$.

$$\begin{array}{r|l} & 2x + 3 \\ \hline x^2+x & 2x^3 + 5x^2 + 15x + 5 \\ & -(2x^3 + 2x^2) \\ \hline & 3x^2 + 15x + 5 \\ & -(3x^2 + 3x) \\ \hline & 12x + 5 \end{array}$$

$$\frac{12x+5}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$$

$$\Rightarrow \quad 12x + 5 = A(x+1) + Bx$$

$$x = 0 \Rightarrow A = 5$$

$$x = -1 \Rightarrow B = 7$$

Thus

$$\int \frac{2x^3+5x^2+15x+5}{x^2+x}\,dx = \int \left(2x + 3 + \frac{5}{x} + \frac{7}{x+1} \right) dx$$

$$= x^2 + 3x + 5\ln|x| + 7\ln|x+1| + C$$

7. (5 points) Determine whether the improper integral $\int_2^\infty \frac{x^2+5}{x^3-7}\,dx$ converges or diverges by making an appropriate comparison.

We have $x^2+5 \geq x^2$ and $x^3-7 \leq x^3$

So $\frac{x^2+5}{x^3-7} \geq \frac{x^2}{x^3} = \frac{1}{x}$.

Since $\int_2^\infty \frac{1}{x}$ diverges, $(p=1)$ $\int_2^\infty \frac{x^2+5}{x^3-7}$ diverges by the Comparison Test.

8. (20 points) Evaluate $\int \frac{x^3}{\sqrt{9+x^2}}\,dx$.

Let $x = 3\tan\theta$

$dx = 3\sec^2\theta\,d\theta$

$\sqrt{9+x^2} = \sqrt{9(1+\tan^2\theta)}$

$= 3\sec\theta$

So we get

$$\int \frac{27\tan^3\theta \cdot 3\sec^2\theta\,d\theta}{3\sec\theta}$$

$$= 27\int \tan^3\theta\sec\theta\,d\theta$$

$$= 27\int (\sec^2\theta - 1)\sec\theta\tan\theta\,d\theta$$

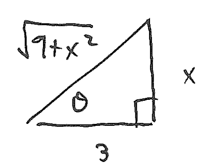
$u = \sec\theta$

$du = \sec\theta\tan\theta\,d\theta$

$$= 27\int (u^2-1)\,du$$

$$= 27\left(\frac{u^3}{3} - u\right) + C$$

$$= 9\sec^3\theta - 27\sec\theta + C$$



$$= 9\left(\frac{\sqrt{9+x^2}}{3}\right)^3 - 27\cdot\frac{\sqrt{9+x^2}}{3} + C$$

$$= \frac{1}{3}(9+x^2)^{3/2} - 9\sqrt{9+x^2} + C$$