MAT 162—Exam #2—10/21/15

Name: Solutions

Show all work using correct mathematical notation. Calculators are not allowed.

1. (6 points) Fill in the initial set-up for applying integration by parts to $\int x^2 \sin 3x \, dx$.

$u =$ x^2 $\qquad dv =$ $\sin 3x \, dx$

$du =$ $2x \, dx$ $\qquad v =$ $-\frac{1}{3} \cos 3x$

2. (4 points) Which of the following is the correct form of the partial fraction decomposition for the function $f(x) = \dfrac{1}{x^3 + 4x^2}$?

(i) $\dfrac{A}{x^2} + \dfrac{B}{x+4}$

(ii) $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x+4}$

(iii) $\dfrac{A}{x} + \dfrac{Bx + C}{x^2 + 4}$

(iv) $\dfrac{A}{x^3} + \dfrac{B}{x^2}$

(v) $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{Cx + D}{x^2 + 4}$

(vi) none of the above

3. (15 points) Evaluate $\int \sin^4 x \cos^3 x \, dx$.

$u = \sin x$

$du = \cos x \, dx$

$$= \int \sin^4 x \, (1 - \sin^2 x) \cos x \, dx$$

$$= \int u^4 (1 - u^2) \, du$$

$$= \int (u^4 - u^6) \, du$$

$$= \frac{1}{5} u^5 - \frac{1}{7} u^7 + C$$

$$= \frac{1}{5} \sin^5 x - \frac{1}{7} \sin^7 x + C$$

4. (12 points) Evaluate $\int x^7 \ln x \, dx$.

IBP :

$u = \ln x \qquad dv = x^7 \, dx$

$du = \frac{1}{x} \, dx \qquad v = \frac{1}{8} x^8$

$$\int x^7 \ln x \, dx = \frac{1}{8} x^8 \ln x - \int \frac{1}{8} x^7 \, dx$$

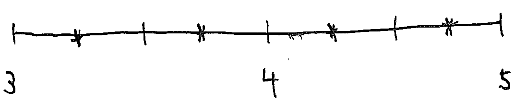
$$= \frac{1}{8} x^8 \ln x - \frac{1}{64} x^8 + C$$

5. (13 points) Consider the integral $\int_3^5 \sqrt{x} \, dx$.

(a) Write out the terms in the Midpoint Rule approximation M_4.

$$\Delta x = \frac{5-3}{4} = \frac{1}{2}$$

$$M_4 = \frac{1}{2} \left(\sqrt{3.25} + \sqrt{3.75} + \sqrt{4.25} + \sqrt{4.75} \right)$$



(b) Find an upper bound for the error when approximating the integral using T_{10}, the Trapezoidal Rule with 10 subintervals.

$f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2} x^{-1/2}$

$f''(x) = -\frac{1}{4} x^{-3/2}$

$|f''(x)| = \frac{1}{4} x^{-3/2} \leq \frac{1}{4 \cdot 3^{3/2}}$ on $[3, 5]$

Thus $|E_T| \leq \dfrac{\frac{1}{12\sqrt{3}} \cdot 2^3}{12 \cdot 10^2}$

6. (10 points) Evaluate $\displaystyle\int_8^{\infty} \frac{dx}{x^{4/3}}$.

$$\int_8^{\infty} \frac{1}{x^{4/3}}\,dx = \lim_{b\to\infty} \int_8^b x^{-4/3}\,dx$$

$$= \lim_{b\to\infty} -3x^{-1/3}\Big|_8^b$$

$$= \lim_{b\to\infty} -3\left(b^{-1/3} - 8^{-1/3}\right)$$

$$= 3\cdot 8^{-1/3}$$

$$= 3/2$$

7. (15 points) Evaluate $\displaystyle\int \frac{1}{\sqrt{x^2+9}}\,dx$.

Let $x = 3\tan\theta$

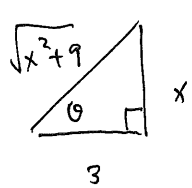
$dx = 3\sec^2\theta\,d\theta$

$\sqrt{x^2+9} = \sqrt{9(\tan^2\theta+1)} = 3\sec\theta$

Then $$\int \frac{1}{\sqrt{x^2+9}}\,dx = \int \frac{3\sec^2\theta\,d\theta}{3\sec\theta}$$

$$= \int \sec\theta\,d\theta$$

$$= \ln|\sec\theta + \tan\theta| + C$$



$$= \ln\left|\frac{\sqrt{x^2+9}}{3} + \frac{x}{3}\right| + C$$

$$= \ln\left|\sqrt{x^2+9} + x\right| + C'$$

8. (7 points) Determine whether the improper integral $\int_1^\infty \frac{\sin^2 x}{x^4+5}\,dx$ converges or diverges. You must clearly state the inequalities used to make a comparison.

We have $\sin^2 x \leq 1$ and $x^4 + 5 \geq x^4$,

So $\frac{\sin^2 x}{x^4+5} \leq \frac{1}{x^4}$

The integral therefore converges by comparison with $\int_1^\infty \frac{1}{x^4}\,dx$ $(p = 4 > 1)$.

9. (18 points) Evaluate $\int \frac{2x+1}{x^3+x}\,dx$.

Partial fractions: $\frac{2x+1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$

$$\Rightarrow \quad 2x+1 = A(x^2+1) + (Bx+c)x$$
$$= (A+B)x^2 + Cx + A$$

$$\Rightarrow \begin{cases} A = 1 \\ C = 2 \\ A+B = 0 \end{cases} \Rightarrow B = -1$$

So $$\int \frac{2x+1}{x^3+x}\,dx = \int \frac{1}{x}\,dx - \int \frac{x}{x^2+1}\,dx + \int \frac{2}{x^2+1}\,dx$$

$\uparrow$

$u = x^2+1$
$du = 2x\,dx$

$$= \ln|x| - \frac{1}{2}\ln(x^2+1) + 2\tan^{-1}x + C$$