MAT 162—Exam #1—9/23/15

Name: Solutions

Show all work using correct mathematical notation. Calculators are not allowed.

1. (10 points) Set up (**but do not evaluate**) a definite integral that gives the area of the surface obtained by revolving the curve $y = e^{3x}$ from $x = 0$ to $x = 2$ about the x-axis.

$f(x) = e^{3x}$

$f'(x) = 3e^{3x}$

$$S = \int_0^2 2\pi e^{3x} \sqrt{1 + (3e^{3x})^2}\, dx$$

$$= 2\pi \int_0^2 e^{3x} \sqrt{1 + 9e^{6x}}\, dx$$

2. (15 points) Find the average value of the function $f(x) = \sin^3 x \cos x$ on the interval $[0, \pi/6]$. Do not leave trigonometric expressions in your final answer.

$$f_{ave} = \frac{1}{\pi/6 - 0} \int_0^{\pi/6} \sin^3 x \cos x\, dx$$

$u = \sin x$

$du = \cos x\, dx$

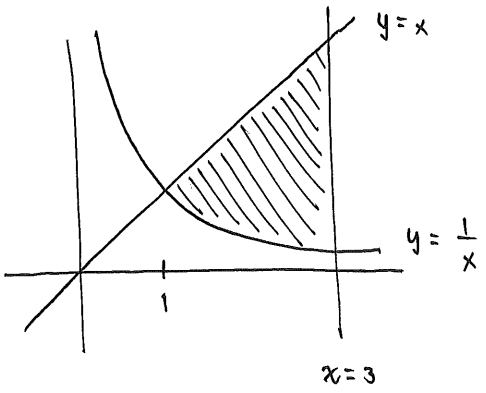
$$= \frac{6}{\pi} \int_0^{1/2} u^3\, du$$

$$= \frac{6}{\pi} \cdot \frac{1}{4} u^4 \Big|_0^{1/2}$$

$$= \frac{3}{2\pi} \cdot \frac{1}{16}$$

$$= \frac{3}{32\pi}$$

3. (15 points) Find the area of the region in the first quadrant bounded by the curves $y = x$, $y = 1/x$, and $x = 3$. Simplify your answer as much as possible.



$$A = \int_1^3 \left(x - \frac{1}{x}\right) dx$$

$$= \frac{x^2}{2} - \ln|x| \Big|_1^3$$

$$= \frac{9}{2} - \ln 3 - \left(\frac{1}{2} - \ln 1\right)$$

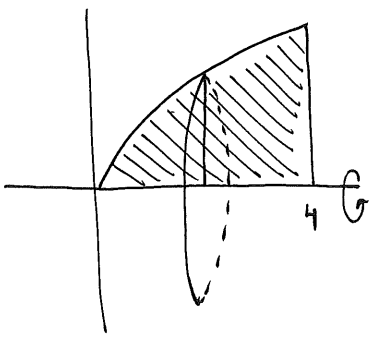
$$= 4 - \ln 3$$

$x = \frac{1}{x}$

$\Rightarrow x^2 = 1$

$\Rightarrow x = \pm 1$

4. (15 points) Find the volume of the solid obtained by revolving the region bounded by the curve $y = x^{1/4}$ and the lines $y = 0$ and $x = 4$ about the x-axis. Simplify your answer as much as possible.



Disks:

$R(x) = x^{1/4}$

$$V = \int_0^4 \pi \left(x^{1/4}\right)^2 dx$$

$$= \pi \int_0^4 x^{1/2} dx$$

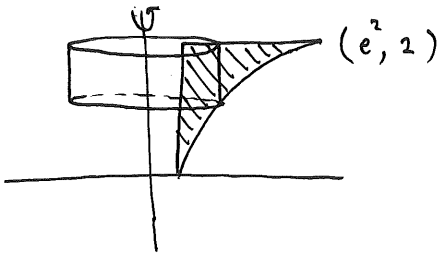
$$= \pi \cdot \frac{2}{3} x^{3/2} \Big|_0^4$$

$$= \pi \cdot \frac{2}{3} \cdot 8$$

$$= \frac{16\pi}{3}$$

5. (15 points) Set up (**but do not evaluate**) definite integrals that give the volumes of the solids obtained by revolving the region bounded by the curves $y = \ln x$, $y = 2$, and $x = 1$ about the given axes. In each case, show a representative disk, washer, or shell on the sketch provided.

(a) the y-axis

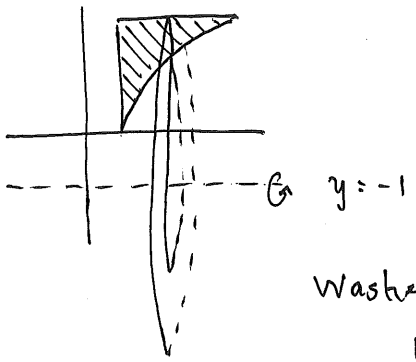


Shells: $r(x) = x$

$h(x) = 2 - \ln x$

$$V = \int_1^{e^2} 2\pi x\,(2 - \ln x)\,dx$$

(b) the line $y = -1$



Washers:

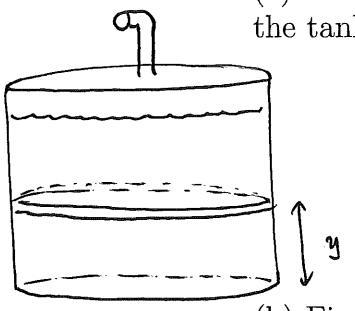
$R(x) = 3$

$r(x) = \ln x + 1$

$$V = \int_1^{e^2} \pi\left(9 - (\ln x + 1)^2\right)dx$$

6. (15 points) A cylindrical tank of radius 8 meters and height 12 meters is filled to a height of 10 meters with water, which weighs 9800 N/m^3. Water is to be pumped out through a spout that extends 3 meters above the tank's top.

(a) Find the weight of a slice of thickness Δy located at y meters from the bottom of the tank.



Volume $= \pi \cdot 8^2 \cdot \Delta y = 64\pi\,\Delta y$

So Weight $= 9800 \cdot 64\pi\,\Delta y$

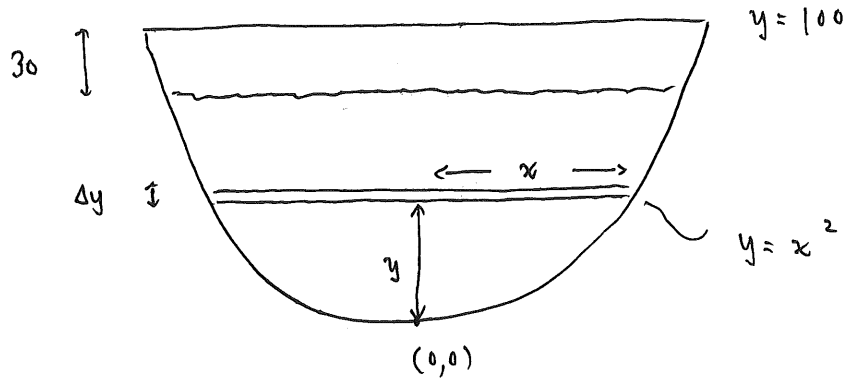
(b) Find the distance moved by the slice discussed in part (a) to reach the top of the spout. You answer should be consistent with the definition of y in the diagram.

Distance $= 15 - y$

(c) Set up (**but do not evaluate**) a definite integral that gives the total work required to empty the tank.

$$\text{Work} = \int_0^{10} 9800 \cdot 64\pi\,(15 - y)\,dy$$

7. (15 points) The wall of a dam is shaped like the region above the parabola $y = x^2$ and below the line $y = 100$. Water weighs 62.4 lb/ft^3, and the water level is 30 feet below the top of the dam.



(a) Find the area of the strip of thickness Δy located at y feet above the bottom of the dam. Your answer should be expressed in terms of the variable y, as labeled in the diagram.

$$\text{Area} = 2x\,\Delta y$$
$$= 2\sqrt{y}\,\Delta y$$

(b) Find the pressure along the strip discussed in part (a).

$$\text{Pressure} = 62.4\,(70 - y)$$

(c) Set up (**but do not evaluate**) a definite integral that gives the hydrostatic force on the dam.

$$\text{Force} = \int_0^{70} 62.4\,(70-y)\cdot 2\sqrt{y}\,dy$$