MAT 161—Sample Final Exam

Name: _____

Calculators are NOT allowed. Show all work using correct mathematical notation.

1. (10 points) Calculate each of the following limits.

   (a) $\lim_{x \to 0} \dfrac{e^x - 1 - x}{x^2}$

   (b) $\lim_{x \to \frac{\pi}{2}} \dfrac{x - 5}{\cos^2 x}$

2. (15 points) Find the derivative of each of the following functions.

   (a) $f(x) = \dfrac{\ln x}{x^3}$

   (b) $g(x) = e^{\sin^{-1} x} \csc 5x$

   (c) $h(x) = \sin^5(\cos^3 x)$

3. (15 points) Evaluate each of the following integrals.

(a) $\displaystyle\int \frac{x^4\,dx}{(x^5+1)^3}$

(b) $\displaystyle\int_0^{\pi/4} \tan^7 x \sec^2 x\,dx$

4. (10 points) Let $A(x) = \displaystyle\int_3^x (t^2-1)^{1/3}\,dt$.

(a) Calculate $A(3)$.

(b) Calculate $A'(3)$.

(c) For what values of x is $A(x)$ increasing?

5. (10 points) Consider the function $y = f(x)$ graphed in Figure 14 on page 88 of the text.

   (a) Evaluate each of the following, or state that it does not exist.

   (i) $\lim_{x \to 3} f(x)$

   (ii) $f(3)$

   (iii) $\lim_{x \to 1^-} f(x)$

   (iv) $\lim_{x \to 5} f(x)$

   (b) At what value(s) of x does f fail to be continuous?

6. (15 points) Sketch the graph of the function $f(x) = x^3 - 6x^2 + 3$, clearly labeling the coordinates of all local extrema and inflection points.

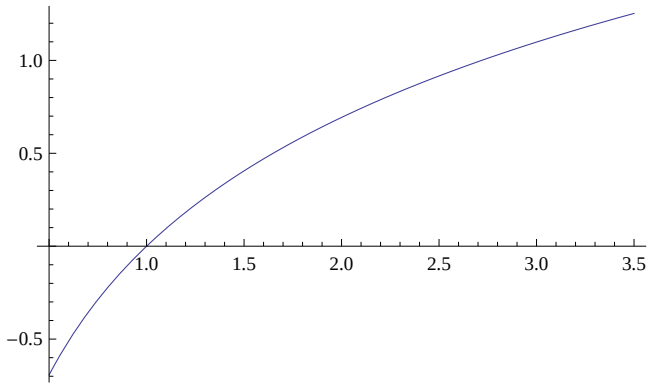
7. (13 points) State the definition of the derivative in terms of a limit, and use it to calculate the derivative of $f(x) = \sqrt{x}$. No credit will be given for shortcut methods.

8. (12 points) Find the equation of the tangent line to the curve $x + y^3 \cos x + \dfrac{4}{y} = 10$ at the point $(0, 2)$.

9. (13 points) A circular oil slick is expanding at a rate of 40 square meters per hour. How fast is its radius increasing at the instant when its area is 100π square meters?

10. (12 points) The acceleration (in m/s^2) of a particle moving along a straight line is given by $a(t) = 12t$. The particle's initial velocity is 10 m/s, and its initial position is 20 meters to the right of the origin. Where is the particle located after 3 seconds?

11. (10 points) Use 4 rectangles with heights determined by the right-hand endpoints to estimate the area under the curve $f(x) = \ln x$ on the interval $[1, 3]$. Show your rectangles on the sketch provided. Do not attempt to add up the terms in your sum.



$R_4 =$ _____ + _____ + _____ + _____

12. (15 points) If 1000 cm^2 of material is available to make a cylindrical can with no top, find the largest possible volume of the can, and justify that your solution gives a maximum.